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A Theoretical Study of Critical Speed of Shaft Carrying a Single Rotor

Suhas S. Jadhav, Sushant S. Jadhav, A. S. Kavitake, A. A. Lawand
(Department of Mechanical Engineering, Modern Education Society's College of Engineering, Pune, India)

Abstract: *Present paper deals with theoretical study of critical speed of shaft carrying single rotor. Every object has its own frequency, called as natural frequency. The measurement of this critical speed and related whirling motion is one of the important problems to be addressed by a design and maintenance engineer. Also the whirling of shaft and its motion comes under the category of self-excited motion i.e. self-excited vibration in which the exciting forces and inducing motion are controlled by the motion itself. When the induced frequency matches with natural frequency, resonance will occur at certain speed, this speed is called as 'critical speed'. In general it is very important to determine the critical speed to avoid the breakdown of rotating machine. By knowing the critical speed we run the machine above or below that critical speed. In this paper we study the critical speed of shaft by the theoretical analysis. The goal of this paper is to present a theoretical understanding of terminology and behavior based in visualizing how a shaft vibrates, and examining issues that affect vibration. It is hoped that this presentation will help the non-specialist better understand what is going on in the machineries, and that the specialist may gain a different view and/or some new examples.*

Keywords : *Critical speed, Whirling shaft, Frequency ratio, Amplitude ratio, Self-excited vibration.*

I. Whirling of Shaft

1.1 Resonance

Resonance is simple to understand if the spring and mass are viewed as energy storage elements – with the mass storing kinetic energy and the spring storing potential energy. when the mass and spring have no external force acting on them they transfer energy back and forth at a rate equal to the natural frequency. In other words, to efficiently pump energy into both mass and spring requires that the energy source feed the energy in at a rate equal to the natural frequency. As in the case of the swing, the force applied need not be high to get large motions, but must just add energy to the system.

The damper, instead of storing energy, dissipates energy. Since the damping force is proportional to the velocity, the more the motion, the more the damper dissipates the energy. Therefore, there is a point when the energy dissipated by the damper equals the energy added by the force. At this point, the system has reached its maximum amplitude and will continue to vibrate at this level as long as the force applied stays the same. If no damping exists, there is nothing to dissipate the energy and, theoretically, the motion will continue to grow into infinity.

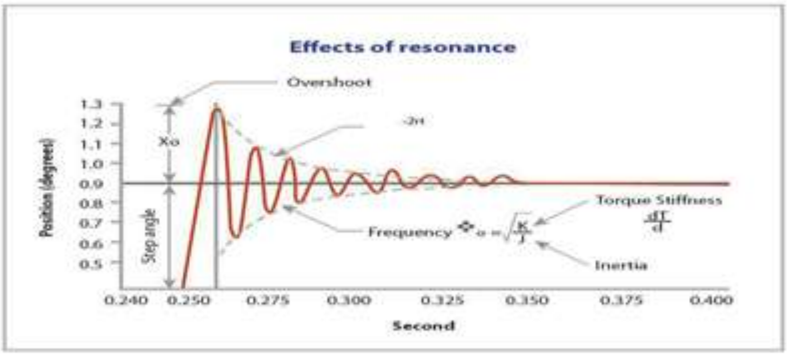


Fig -1 Resonance

1.2 Critical speed

All rotating shafts, even in the absence of external load, deflect during rotation due to self-weight. The combined weight of a shaft and shaft-mountings can cause deflection that will create resonant vibration at some speed. These speeds are commonly known as critical speed. Shaft deflection depends upon the followings:-

a) Material Stiffness and number of supports
b) Self Weight and mountings
c) Unbalanced centrifugal forces
d) Lubricant viscosity

In solid mechanics, in the field of rotor dynamics, the critical speed is the theoretical angular velocity that excites the natural frequency of a rotating object, such as a shaft, propeller , lead screw, or gear. As the speed of rotation approaches the object's natural frequency, the object begins to resonate, which dramatically increases system vibration. The resulting resonance occurs regardless of orientation. The rotational speed is equal to the numerical value of the natural vibration, then that speed is referred to as critical speed. There are two main methods used to calculate critical speed—the Rayleigh-Ritz method and Dunkerley's method. Both calculate an approximation of the first natural frequency of vibration, which is assumed to be nearly equal to the critical speed of rotation.

In general Ritz and Dunkerley‘s equation overestimates and the Dunkerley‘s equation underestimates the natural frequency. The equation illustrated below is the Ritz and Dunkerley‘s equation, good practice suggests that the maximum.

When the speed of rotation is increased the centrifugal force also increases and so does the restoring force. At low critical speeds, there is increase the restoring forces with the increase in shaft deflection. Shaft deflection is unchecked and the shaft behaves as a flexible element operation speed should Rotating shafts tend to deflect transversely during rotation even in the absence of external load due to weight of the shaft itself. The combined weight of rotor/wheel on the shaft and the shaft may cause unbalancing in the shaft due to centrifugal force. This centrifugal force induces vibration in the shaft. These vibrations become very large when the shaft rotates at a speed equal to the natural frequency of the transverse oscillation. This phenomenon is known a whirling of the shaft. Whirling can be very damaging to the rotors and hence to the machinery such as turbine generator which can stop the power production due to shaft break down. The system must be carefully balanced to reduce or avoid whirling effect and should be designed to have different natural frequency other than the operating speed of rotation. Whirling also damages the bearing and turbine blades during starting and stopping of the machinery.

II. Critical Speed Of Shaft Carrying Single Rotor (Without Damping)

A vertical shaft having negligible inertia and carrying a single rotor, the shaft in stationary condition, When shaft is in rotating condition as shown in ,then there are two forces acting on the shaft:

1. Centrifugal Force = mw^2 (y +e)

It acts in radially outward direction through point G

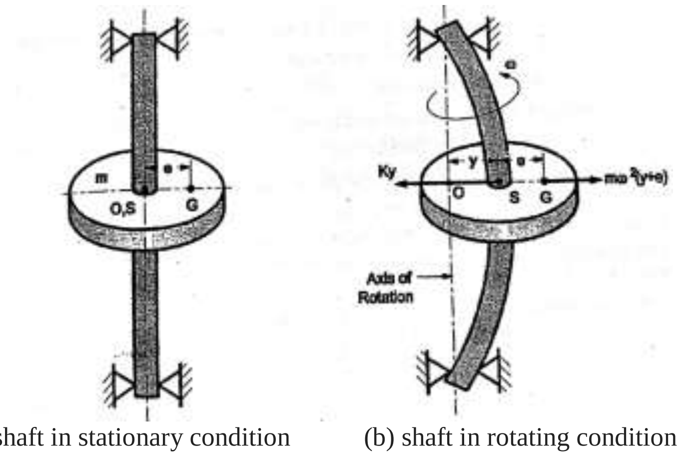
2. Restoring Force = Ky

It acts in radially inward direction through point G.

3. In equilibrium condition, the centrifugal force is equal to restoring force. Therefore, Centrifugal force = Restoring force

$$y = \frac{e*(\frac{\omega}{\omega_c})^2}{1-(\frac{\omega}{\omega_c})^2} \quad \frac{e*(\frac{\omega}{\omega_c})^2}{1-(\frac{\omega}{\omega_c})^2}$$

From Equation, it is clear that, as the angular speed of the shaft w' increases, the deflection of the shaft 'y' increases. When 'u' becomes equal to 'w_n', the deflection of the shaft y becomes infinity,



(a).shaft in stationary condition (b) shaft in rotating condition

Fig-2 Whirling Of Shaft

Thus, speed at which the defection of tends to be infinity is known as critical speed or whirling speed.

2.1 Ranges Of Shaft Speed-

There are three ranges of shaft speed ‘ω’

1. Shaft speed (ω)< Critical speed (ω_c)**:**

When the speed of shaft is less than the critical speed (i.e. $\omega<\omega_c$), the deflection of shaft ‘y’ is positive. This means, the rotor rotates with heavy side outwards.

In this speed range, the deflection of shaft 'y' increases with shaft speed ‘ω’

2. Shaft speed (ω) = Critical speed (ω_c)**:**

when the speed of shaft is equal to the critical speed (ie, $\omega=\omega_c$), the deflection of shaft ‘y’ tends to be infinity and the shaft vibrates with large amplitude. This may lead to the failure of the shaft.

3. Shaft speed (ω)>Critical speed (ω_c)**:**

When the speed of shaft is greater than the critical speed (i.e. $\omega>\omega_c$), the deflection of shaft ‘y' is negative.

In this speed range, the deflection of shaft 'y' and eccentricity 'e' are on the same side of the geometric centre of the rotor ‘s’. This means, the rotor rotates with light side outwards. When $\omega >>\omega_{c,}$ y= - e which means that the centre of gravity of rotor 'G' approaches the axis of rotation 'O' and the rotor rotates about its C.G. This principle is used in running high speed turbines by speeding up the rotor rapidly beyond the critical speed. When 'y' approaches the value of -e', the rotor runs steadily.

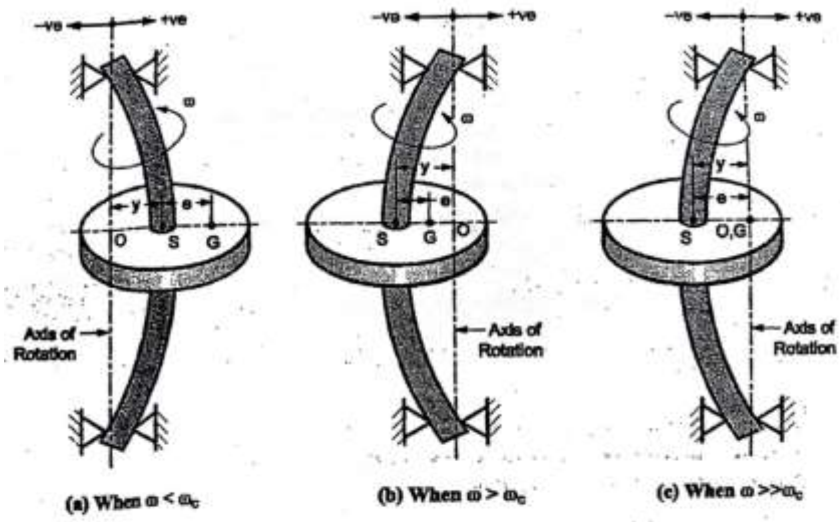


Fig-3 Ranges of shaft speed

III. Calculations For Critical Speed

In theoretical measurement of critical speed of rotating shaft 8 mm & 6 mm diameter.

Known Parameters :-

Shaft material – Mild Steel

Density :- 7.87 gm/cm^3 = 7.87 * 10^3 kg/m^3

Modulus of elasticity :- E :- 200 GPa.

Diameter of shaft :- d :- 8 mm = 0.008 m

Length of shaft :- L :- 92 cm = 0.92 m

Theoretical critical speed analysis :-

Mass of Shaft per metre length,

Mass = volume * density

= Area * length * density

Area of shaft = $\frac{\pi}{4}\,d^2\frac{\pi}{4}\,d^2$

= $\frac{\pi}{4}\,(0.008)^2\frac{\pi}{4}\,(0.008)^2$

= $5.0265 * 10^{-5}$ m^2 ……………………..1

From equation 1,

M_s = $5.0265 * 10^{-5} * 0.92 * 7.87 * 10^{3}$

= $363.9421 * 10^{-3}$ kg/m ……………………….2

Mass of rotor (m_r) = 0.150 kg

M = $M_s + m_r$

= 0.5139 kg/m ……………………………3

Static deflection due to point load for the both ends are fixed,

$$\delta = \frac{WL^3 \quad WL^3}{192\ EI192\ EI} \text{ m}$$

where I = mass moment of inertia

$$= \frac{\pi}{64} d^4 \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (0.008)^4 \frac{\pi}{64} (0.008)^4$$

= $20106 * 10^{-10}$ m^4 …………………………..4

W = M*g

= 0.5139 * 9.81

= 5.0413 N ……………………………. 5

$$\delta = \frac{5.0413*(0.92)^3 \qquad 5.0413*(0.92)^3}{192*200*10^9*2.0106*10^{-10}192*200*10^9*2.0106*10^{-10}}$$

= $5.0845 * 10^{-4}$ m ………………………...6

As per Dunkerley's Formula,

$$F_c = F_n = \frac{0.4985 \quad 0.4985}{\sqrt{\delta+\frac{\delta_s}{1.27}}\sqrt{\delta+\frac{\delta_s}{1.27}}} \text{ Hz}$$

$$= \frac{0.4985 \qquad 0.4985}{\sqrt{5.0845*10^{-4}+\frac{0}{1.27}}\sqrt{5.0845*10^{-4}+\frac{0}{1.27}}} \text{ Hz[5]}$$

(where $\delta_s \delta_s$ = 0, there is no UDL on the shaft.)

= 22.1079 Hz ……………………………..7

$\omega_c \omega_c$ = 2π F_c

= 2π * 22.09

= 138.7955 rad/s

$$\omega_c = \frac{2\pi N_c}{60}$$

Where N_c = whirling speed of shaft/ critical speed of shaft

$$138.7955 = \frac{2\pi N_c}{60}$$

$N_{c\ (theoretical)}$= 1326.4749 rpm……Answer

On experimental setup Critical speed measured with the help of tachometer :-

$N_{c\ (experimental)}$ = 1101 rpm

For 8 mm shaft, Rotor mass changed from 0.300 kg and 0.350 kg, the theoretical critical speed is 1167.09188 rpm and 1125.4909 rpm respectively.

Similarly, for 5 mm, 6 mm, 8 mm, 10 mm & 12 mm diameter of shaft and 0.150 kg, 0.300 kg & 0.350 kg respectively.

Following are the theoretical critical speed observations for different shaft diameters and rotors:

Table -1: theoretical critical speed observations for different shaft diameters and rotors

Density (kg/m^3)	modulus E (Gpa)	Length (m)	Rotor mass (kg)	Diameter (m)	Frequency critical (Hz)	Theoretical critical speed (rpm)
7870	2E+11	0.92	0.15	0.005	11.4543	687.2607
7870	2E+11	0.92	0.3	0.005	9.31122	558.6732
7870	2E+11	0.92	0.35	0.005	8.8256	529.5385
7870	2E+11	0.92	0.15	0.006	14.9691	898.1500
7870	2E+11	0.92	0.3	0.006	12.5495	752.9757
7870	2E+11	0.92	0.35	0.006	11.9707	718.2447
7870	2E+11	0.92	0.15	0.008	22.1079	1326.4749
7870	2E+11	0.92	0.3	0.008	19.4515	1167.0918
7870	2E+11	0.92	0.35	0.008	18.7581	1125.4909
7870	2E+11	0.92	0.15	0.01	29.2116	1752.6989
7870	2E+11	0.92	0.3	0.01	26.5708	1594.2487
7870	2E+11	0.92	0.35	0.01	25.8377	1550.2676
7870	2E+11	0.92	0.15	0.012	36.2278	2173.6719
7870	2E+11	0.92	0.3	0.012	33.7128	2022.7715
7870	2E+11	0.92	0.35	0.012	32.9841	1979.0478

Observations have been plotted Figure 4 and 5 which shows that decrease in critical frequency as well as critical speed rpm with increase in rotor mass for various shaft diameters.

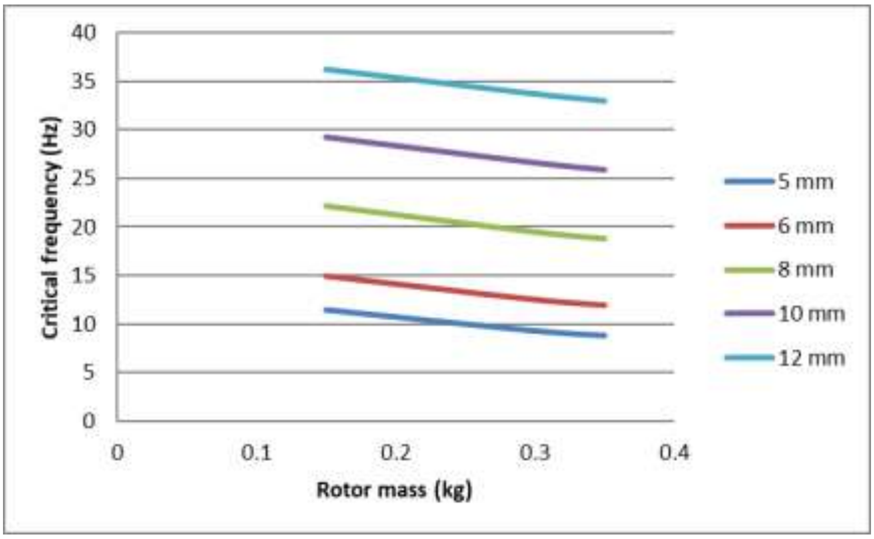


Fig. 4: Theoretical Critical speed (rpm) versus Rotor mass (kg) for different shaft diameter

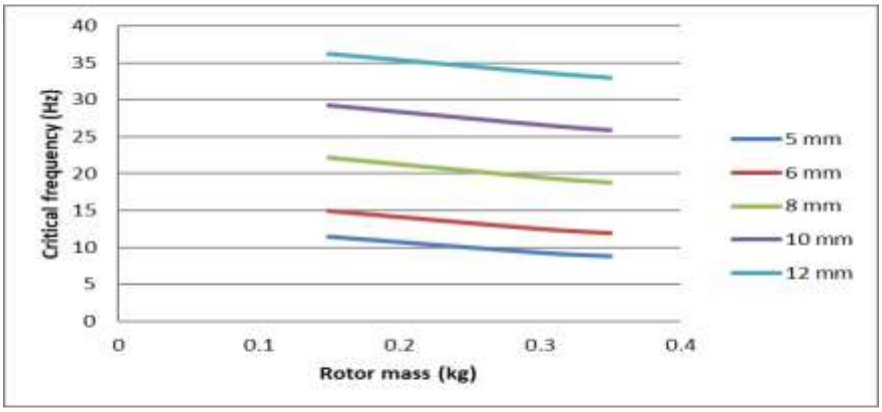


Fig. 5: Critical Frequency (Hz) versus Rotor mass (kg) for different shaft diameter

IV. Conclusion

we can conclude that, as the diameter of shaft increases the critical speed of shaft also increases. The relationship between critical speed and diameter of shaft is that the critical speed rapidly increases with increase in the shaft diameter provided that the mass of rotor is kept constant. Keeping the diameter of shaft constant, the theoretical speed of shaft decreases with increase in the mass of the rotor. Also there is decrease in critical frequency as well as critical speed rpm with increase in rotor mass.

Conflict of interest The authors declare that there is no conflict of interests regarding the publication of this paper.

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